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intuitive geomery. It is semi-experimental. On an intuitive basis it erects a logical edifice under the constant supervision of the architect Intuition.

East High School,
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SOME THOUGHTS ON SPACE.

By E. D. Roe, Jr.

The statement is frequently made that Kant's theory of space has been overthrown.

The question at issue is only the old battle as to whether all our knowledge is derived from sensation or not,* renewed on the field of geometry. After an armistice, the fortifications built by Kant have been bombarded by the modern heavy and light artillery and rapid fire guns of empirical psychology, and while some defects have been disclosed in the walls, it is pertinent to look and see after the smoke of battle has cleared, if the walls are razed to the ground.

Doubtless to the extreme to which Kant identified his theory with the Euclidean form of space a correction needs to be made. But the question still arises, would this be the case without this identification, and is his view overthrown *in toto?*

It is the object of this communication to raise this question, and ask if any modification of his theory can be proposed which leaves any part of it still in force. It has seemed to the writer that the following considerations might be suggested.

The fundamental principle of Kant was that whatever experience† occurs (to a finite reason) must occur in time, and, if external, must occur in time and space. Time and space are necessary conditions of the possibility of any experience whatever. It would seem that these principles are not overthrown, nor can they be overthrown, least of all by experience, for which they are the condition of possibility. Thus it can be said prior to or before the event, if the sun rises tomorrow morning, it must rise in time and space.

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* The way in which Leibnitz touched up Aristotle on this question is as good to day as ever. Says Leibnitz: "Hence, to the well-known adage of Aristotle, 'nihil est in intellectu quod non fuit prius in sensu,' I have added this qualification—nisi intellectus ipse," an exception which is almost the whole thing, and which shows that this seven-headed thinker (Leibnitz) who preceded Kant anticipated the Kantian theory as he did many another discovery, for his exception to or revision of Aristotle is the Kantian theory in a nut shell.

† What Kant means by experience is a concatenated experience. A mere experience, or stream of sensations could be had in a consciousness without the categories and intuitions. They would not be necessary as a condition of its possibility. But this is not the kind of

But Kant went farther and held that not only is space in general the necessary condition for the possibility of the experience of external objects, but indeed a Euclidean space. If the sun rises tomorrow morning it must rise in time and in a Euclidean space.

His error then consisted in identifying the general form of space with a particular form, 'the Euclidean. If he had not tried to state the particular form as necessary, it seems that he would have been so far unassailable.

The question next arises why do we assume the Euclidean when the mind has at least two other forms which would account for the facts equally well. Possibly experience has had some influence, but first of all experience has never shown us and probably never can show us that we are in a Euclidean space. How then has it influenced us in a matter of which it is acknowledged that it is impotent to be the arbiter? Is there anything else which has influenced us? It would seem that reason in its unconscious but correct working has influenced us.

The Euclidean form is the simplest form which the mind has, and in the course of the mind's childhood, was the easier to grasp and apply. By maturer reflection it was discovered by a process of pure reasoning, not by experiments on nature, that the mind had other forms of space, of which two at least, the hyperbolic and the elliptic forms, would, with proper attention to the parameters have applied to our space. In fact all three approach the same limit, as the parameter becomes indefinitely great. And all three may be regarded as special forms of a general form which includes in itself all three. In not applying the hyperbolic and elliptic forms, the mind unconsciously and

experience which the theory of knowledge is called upon to explain. The categories and forms of intuition are necessary to the possibility of such a rational and rationally connected experience as we find ours in consciousness to be.

correctly followed a logical demand (of its own creation) that of alternative or possible hypotheses to explain an experience the simplest is to be chosen.

Thus we have not derived the knowledge of other forms than the Euclidean from experience, and as these and the Euclidean can be regarded as particular forms of a general form, if we say Euclidean space is derived from experience, we have to explain how it is that the particular is, while the general which contains it and presupposes it is not, derived from experience. Moreover, all experiments conducted with a view to determine whether space is derived from experience are vitiated by the fact that the exact knowledge of space has to be presupposed before the experiment, in other words the experiment is superfluous.

That Kant posited the Euclidean form as necessary was an inadvertance for which he is not to be severely blamed, as the other forms had not been matured by reason in his day. His error is somewhat analogous to the errors made in analysis, where in old time infinite divergent series were used by many celebrated worthies. A sharper criticism showed the unsoundness of this use, but did not show that analysis was therefore of empirical origin. It simply showed that a mistake had been made and was corrected. *It vindicated analysis.* If the error in regard to space had been detected by empirical means, it would certainly have to be yielded that space is of empirical origin. But the very means used to detect the error show that several forms of space are at least created by the mind (they or some of them may also exist externally to it) and that among them as a special case is the Euclidean form. The exact perfection of all these forms shows that they are at least creatures of reason. Whether they have also objective existence is beyond a proof. Either their ideality or their objectivity lies in the land of belief, and the proper instrument for investigating this is the calculus of probabilities in its widest logical significance.

Compare their sharpness and exactness with any experimental or physiological fact. What idea at all have we of a color? None. It must be experienced, in order to be appreciated. But its effect is only in the sensation and with the sensation it is gone. What idea have we of weight? None. We can determine its effects in time and space, to be sure. It is an intensive magnitude, and in so far as the assumed effects of intensive magnitudes appear in time and space, both the time and space appearance can be measured. But what is it itself? What idea have we of it?

In the case of a Euclidean triangle the idea is perfectly sharp. The conclusions from Euclidean premises are universal and necessary from the premises. Experience gives no such results. To say that, in demonstrating the theorem that the sum of the three interior angles of a Euclidean triangle is equal to two right angles, we are performing an experiment, is to trifle with words and misuse language. No experiment on nature is performed. No question is put to nature. Reason is only concerned in tracing out necessary connections between its own creations. It may be an experiment in a pedagogical sense whether a given student will prove the proposition, but this is a confusion of ideas and a throwing of dust in our faces in the place of argument. The real question is not whether this particular student will now prove it, ***but can it be proved by reason.*** And it is maintained here that this is the only way that it ever has been or ever can be proved. An experimental proof in the strict sense would be no proof at all. Nothing but a more or less close approximation would be obtained and obscurity and darkness would replace light.

In saying that time and space are ***a priori*** forms of intuition which are necessary to the possibility of experience, it is not meant that time and space are innate ideas (Descartes) fully developed in the mind before experience occurs, but only that when experience does occur and the mind is stimulated by something not of itself, it will act according to certain forms on the stimulus, and not otherwise, that is according to necessary forms of its own action. It can not act otherwise if it act at all. It possesses potentially the forms, but not actually before experience. And in its practical action only such as are practically necessary for its needs will be called out and applied. Thus the hyperbolic and elliptic forms were not needed for its practical ends and were only called out later by an introspective process. As far as time is concerned, the forms of space first appear at the time of experience. Experience is the occasion of their being brought out and applied. If no experience happened they would never come forth. We might reverse the Kantian dictum and say experience is necessary to the possibility of the appearance in consciousness of the forms of time and space as well as of the categories. And Kant himself maintained this principle by implication.

The point however is that the forms of intuition and the categories were not something communicated to consciousness or handed over by experience. In fact the possibility of this communication would be inexplicable, if we did not first presuppose an understanding capable of receiving the communication, that is capable of itself forming the very ideas supposed to be communicated. And if it can itself form the ideas, and does so on the occasion of experience, here again the communication is superfluous, and again the simplest hypothesis is not that a handing over to a blank, which is unintelligible, took place, but that reason by its own spontaneity, at least, acted according to the laws of its own activity, and applied its forms and categories to the matter of experience.

It seems to me that the empirical psychologists are always concerned with a refined investigation of the empirical occasions on which the form of space is called into being as a response by the mind to the external occasions. As to these empirical occasions there is no necessary order. They may be highly variable in different cases, and they are of course empirical. But it is maintained the different empirical occasions are not space, nor the thought of it. They confuse these occasions with space, and thus believe that they have attached to space an empiricality which only belongs to the occasions on which the mind puts forth its form in consciousness.

If the mind is not constituted so to act space will not result no matter how much experience is had. A cow has the same empirical occasions, but does any one imagine that mathematical space is in a cow's mind, or ever will be, or that it could be revealed to or handed over to the cow, or by any instruction be conveyed to or gotten into the cow's consciousness? If experience could cause its genesis it would be there, for a cow has as much expereince as we, doubtless more, because the cow has nothing but experience and all of it in time and space.

A honey bee builds a comb which could only have been consciously deduced by an expert mathematician, but has the honey bee derived it from reason or is it conscious of it? It would seem that here is not an innate idea but an innate instinct. The bee builds without being conscious of the complexity of the space relations or the complexity of the building. But if experience were the cause of the space concept, the honey bee, above all, should have it. But it gives no evidence of this, since for centuries it has built with dumb monotony the same invariable cell. Thus we see a mind is presupposed which is capable of itself forming the notion, in order that the notion may appear.

This does not prove that time and space are not objective realities also. Kant denied their objective reality. But he should not have done so as by his theory he did not know what was external to the mind. He should have neither denied nor asserted this. In fact so far as it is left to pure reason to decide, neither the ideality on the one hand, nor the objectivity on the other hand of time and space can be established or disestablished, but they are as we said before a concern of probability, a question for the practical reason.

In conclusion then, it does not seem that Kant's fundamental principle that time and space in general are necessary forms of intuition is overthrown, but that the mind acting by its own spontaneity applies its categories and forms of intuition to the matter of experience, and that in the way it has come to apply the Euclidean form it has not received a revelation from experience, nor can it probably receive such a one (the equally applicable hyperbolic and elliptic forms stand in the way of this) that our space is Euclidean, but rather has it chosen, according to its own laws and logical demands, to apply the simplest theory, the Euclidean theory, as entirely competent to its practical needs with reference to our space.

Finally our space may also be in fact Euclidean, and objectively exist, but probably this can neither be proved nor disproved. In fact if some measurements or observations in the visible universe would be better reconciled by a hyperbolic or elliptic space, such observations would demand most careful reinvestigation and adjustment. Still they would not prove that the mind has not these forms prior to their communication by experience, since we already have them, only we have not as yet discovered a practical need for their application.

Until such discoveries are made it is not scientific to use the thought of them for argument.

The preceding conclusion was reached independently. My attention has since been called to the book of Dr. Carus, "The Foundations of Geometry," Chicago, The Open Court Publishing Co., 1908. It is a great pleasure to recognize in him a friend and not an antagonist. The results here reached seem to agree with those of Dr. Carus, though the method and standpoint are a little different. What he calls "the apriority of motion" I should call one of the empirical occasions on which the mind by its spontaneity acts and brings into consciousness the form of space. It would seem that motion is not the only empirical occasion on which the form is called forth. Why might not one lie perfectly still with eyes closed and receive tactile sensations on different parts of his body and some notion of here and there be called out without the necessity of motion? I should replace his "abstraction" by spontaneity. He says space is not the product of pure reason. I should say it is, but not of pure reason in its discursive or logical activity by the application of the categories. That is I should maintain that pure reason has an activity outside of its logical action, and that this is not categorical action, but intuitive action. It is Kant's "Anschauung." That is pure reason has a faculty of pure thought and a faculty of pure intuition. This would merely be a question of definition of pure reason. These are some differences of view and statement which can not divide a house against itself.

If I were asked my reasons for preferring spontaneity to "abstractions," I should say: If the "abstractions" are abstractions from experience they are only general inductive or empirical laws and as such can never have the character of necessity or universality. This very theory has been offered by the opposite side, to explain away the universality and necessity claimed for the categories and forms of intuition, and I should not want to seem to favor it. Again to most persons I fear the word abstraction conveys an idea of unreality. "It is a mere abstraction." Therefore for the same reason that I would avoid the use of "imaginary" for "complex number," I would avoid a term which might cause a weakening of the thought to be conveyed. For me spontaneity seems to be a positive word and to call up the thought of lively reality in action.

I also take pleasure in referring to a paper of Professor W. H. Metzler, "Geometry," *Secondary Education,* Bulletin 28, Albany, N. Y., 1905, pp. 47–54, in which he arrived at final results in harmony with those of Dr. Carus and the writer, that geometry is not an empirical physical science, but a pure science.

SYRACUSE UNIVERSITY, May 27, 1909.